POSNACK
SCHOOL

Geometry– Summer Math Packet
Student Information Page

We're so proud of you for taking the time to work on math over the summer!

Here are some helpful hints for success:

- ☺ Find a quiet work space where you can get organized and stay focused.
- ☺ Pay close attention to the examples and vocabulary.
- ☺ Choose a unit and work through it completely before moving on to the next unit.
  - Complete all of the problems on each worksheet.
- ☺ Remember to do a little work each week. DO NOT wait until the week before school starts to complete your packet!

☺ The packet should be returned to your math teacher at the end of the first week of school.

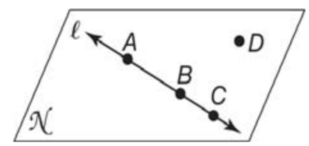
Have fun & we'll see you in August!

Geometry Honors
Summer Work
Chapter 1 Tools of Geometry
1-1 Points, Lines, and Planes

Name Points, Lines, and Planes In geometry, a **point** is a location, a **line** contains points, and a **plane** is a flat surface that contains points and lines. If points are on the same line, they are **collinear**. If points on are the same plane, they are **coplanar**.

Example: Use the figure to name each of the following.

a. a line containing point *A*

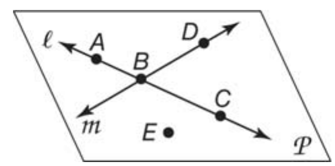


The line can be named as ℓ. Also, any two of the three points on the line can be used to name it.
$\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, or $\overleftrightarrow{BC}$

b. a plane containing point *D*
The plane can be named as plane *N* or can be named using three noncollinear points in the plane, such as plane *ABD*, plane *ACD*, and so on.

Exercises

Refer to the figure.



1. Name a line that contains point *A*.

2. What is another name for line *m*?

3. Name a point not on $\overleftrightarrow{AC}$.

4. What is another name for line ℓ?

5. Name a point not on line ℓ or line *m*.

Draw and label a figure for each relationship.

6. $\overleftrightarrow{AB}$ is in plane *Q*.

7. $\overleftrightarrow{ST}$ intersects $\overleftrightarrow{AB}$ at *P*.

8. Point *X* is collinear with points *A* and *P*.

9. Point *Y* is not collinear with points *T* and *P*.

10. Line ℓ contains points *X* and *Y*.

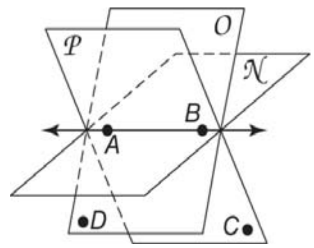
1-1 Points, Lines, and Planes (continued)

Points, Lines, and Planes in Space: **Space** is a boundless, three-dimensional set of all points. It contains lines and planes. The **intersection** of two or more geometric figures is the set of points they have in common.

Example

a. Name the intersection of the planes *O* and *𝒩*.
The planes intersect at line $\overleftrightarrow{AB}$.

b. Does $\overleftrightarrow{AB}$ intersect point *D*? Explain.
No. $\overleftrightarrow{AB}$ is coplanar with *D*, but *D* is not on the line $\overleftrightarrow{AB}$.

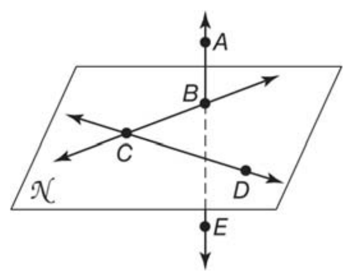


Exercises
Refer to the figure.

1. Name the intersection of plane *N* and line $\overleftrightarrow{AE}$.

2. Name the intersection of $\overleftrightarrow{BC}$ and $\overleftrightarrow{DC}$.

3. Does $\overleftrightarrow{DC}$ intersect $\overleftrightarrow{AE}$? Explain.

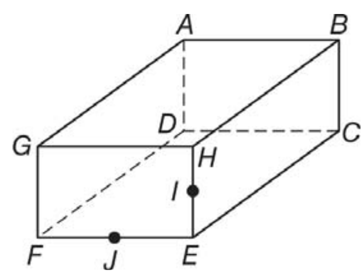


Refer to the figure.

4. Name the three line segments that intersect at point *A*.

5. Name the line of intersection of planes *GAB* and *FEH*.

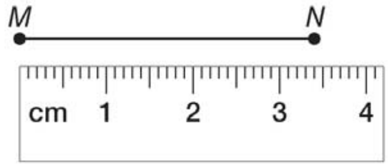
6. Do planes *GFE* and *HBC* intersect? Explain.



1-2 Linear Measure

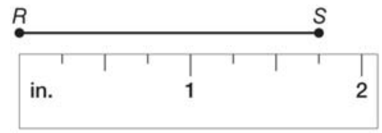
Measure Line Segments A part of a line between two endpoints is called a **line segment.** The lengths of $\overline{MN}$ and $\overline{RS}$ are written as *MN* and *RS*. All measurements are approximations dependent upon the smallest unit of measure available on the measuring instrument.

Example 1: Find the length of $\overline{MN}$.



The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter. The length of $\overline{MN}$ is about 34 millimeters.

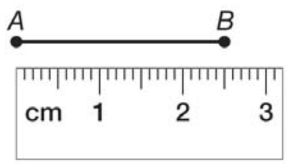
Example 2: Find the length of $\overline{RS}$.



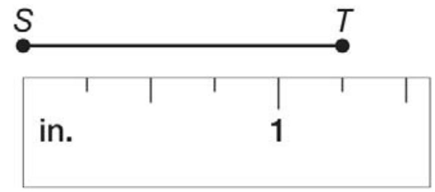
The long marks are inches and the short marks are quarter inches. Point *S* is closer to the $1\frac{3}{4}$ inch mark. The length of *RS* is about $1\frac{3}{4}$ inches

Exercises
Find the length of each line segment or object.

1.



2.



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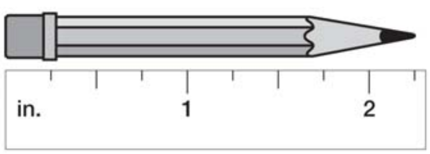

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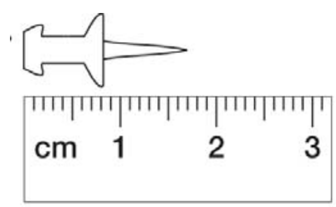

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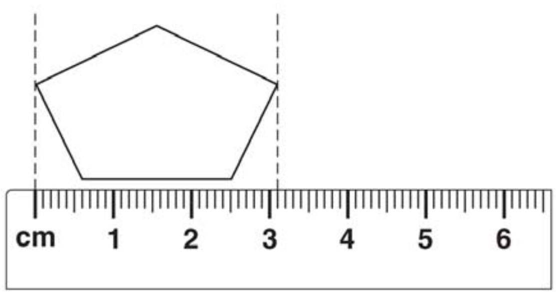
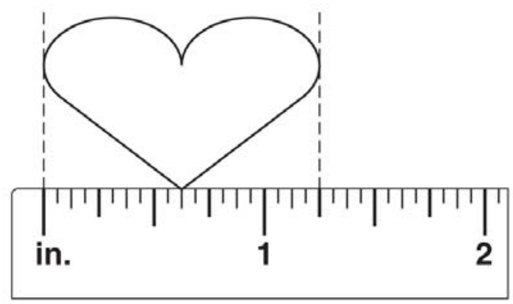
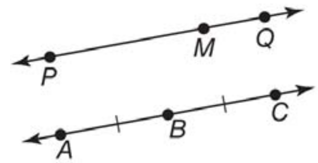


6.



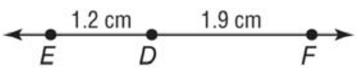
1-2 Linear Measure (continued)



Calculate Measures On $\overleftrightarrow{PQ}$, to say that point *M* is between points *P* and *Q* means *P*, *Q*, and *M* are collinear and $PM + MQ = PQ$.

On $\overleftrightarrow{AC}$, $AB = BC = 3$ cm. We can say that the segments are **congruent segments**, or $\overline{AB} \cong \overline{BC}$. Slashes on the figure indicate which segments are congruent.

Example 1: Find *EF*.

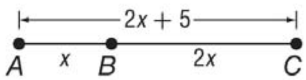


Point *D* is between *E* and *F*. Calculate *EF* by adding *ED* and *DF*.

$ED + DF = EF$	Betweenness of points
$1.2 + 1.9 = EF$	Substitution
$3.1 = EF$	Simplify.

Therefore, $\overline{EF}$ is 3.1 centimeters long.

Example 2: Find *x* and *AC*.



B is between *A* and *C*.

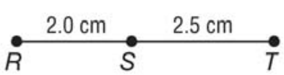
$AB + BC = AC$	Betweenness of points
$x + 2x = 2x + 5$	Substitution
$3x = 2x + 5$	Add $x + 2x$.
$x = 5$	Simplify.

$AC = 2x + 5 = 2(5) + 5 = 15$

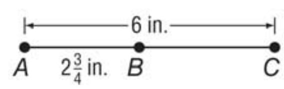
Exercises

Find the measurement of each segment. Assume that each figure is not drawn to scale.

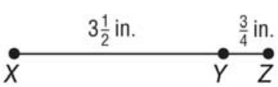
1. $\overline{RT}$



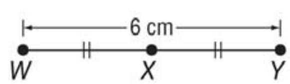
2. $\overline{BC}$



3. $\overline{XZ}$



4. $\overline{WX}$



ALGEBRA Find the value of *x* and *RS* if *S* is between *R* and *T*.

5. $RS = 5x$, $ST = 3x$, and $RT = 48$

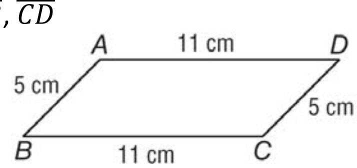
6. $RS = 2x$, $ST = 5x + 4$, and $RT = 32$

7. $RS = 6x$, $ST = 12$, and $RT = 72$

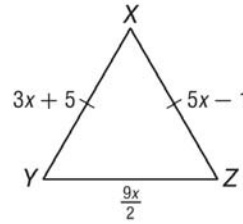
8. $RS = 4x$, $ST = 4x$, and $RT = 24$

Determine whether each pair of segments is congruent.

9. $\overline{AB}$, $\overline{CD}$

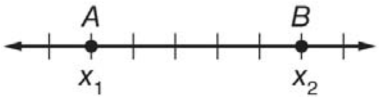
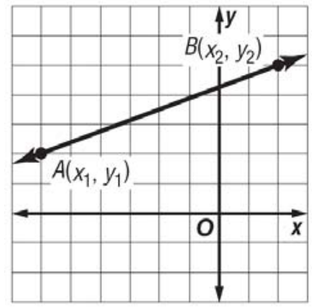


10. $\overline{XY}$, $\overline{YZ}$

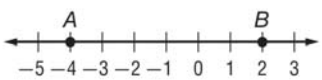


1-3 Distance and Midpoints

Distance Between Two Points

Distance on a Number Line	Distance in the Coordinate Plane
 $AB = \|x_1 - x_2\|$ or $\|x_2 - x_1\|$	Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 

Example 1: **Use the number line to find *AB*.**



$AB = |(-4) - 2|$

$= |-6|$

$= 6$

Example 2: **Find the distance between *A*(–2, –1) and *B*(1, 3). Distance Formula**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(1 - (-2))^2 + (3 - (-1))^2}$

$AB = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{25}$

$= 5$

Exercises

Use the number line to find each measure.

1. *BD*

2. *DG*

3. *AF*

4. *EF*

5. *BG*

6. *AG*

7. *BE*

8. *DE*

Find the distance between each pair of points.

9. *A*(0, 0), *B*(6, 8)

10. *R*(–2, 3), *S*(3, 15)

11. *M*(1, –2), *N*(9, 13)

12. *E*(–12, 2), *F*(–9, 6)

13. *X*(0, 0), *Y*(15, 20)

14. *O*(–12, 0), *P*(–8, 3)

15. *C*(11, –12), *D*(6, 2)

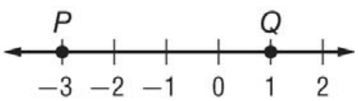
16. *K*(–2, 10), *L*(–4, 3)

1-3 Distance and Midpoints (continued)

Midpoint of a Segment

Midpoint on a Number Line	If the coordinates of the endpoints of a segment are x_1 and x_2, then the coordinate of the midpoint of the segment is $\frac{x_1 + x_2}{2}$.
Midpoint on a Coordinate Plane	If a segment has endpoints with coordinates (x_1, y_1) and (x_2, y_2), then the coordinates of the midpoint of the segment are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Example 1: Find the coordinate of the midpoint of $\overline{PQ}$.

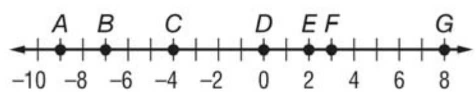


The coordinates of *P* and *Q* are –3 and 1.
If *M* is the midpoint of $\overline{PQ}$, then the coordinate of *M* is $\frac{-3 + 1}{2} = \frac{-2}{2}$ or –1.

Example 2: Find the coordinates of *M*, the midpoint of $\overline{PQ}$, for *P*(–2, 4) and *Q*(4, 1).
$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 4}{2}, \frac{4 + 1}{2}\right)$ or (1, 2.5)

Exercises

Use the number line to find the coordinate of the midpoint of each segment.



1. $\overline{CE}$

2. $\overline{DG}$

3. $\overline{AF}$

4. $\overline{EG}$

5. $\overline{AB}$

6. $\overline{BG}$

7. $\overline{BD}$

8. $\overline{DE}$

Find the coordinates of the midpoint of a segment with the given endpoints.

9. *A*(0, 0), *B*(12, 8)

10. *R*(–12, 8), *S*(6, 12)

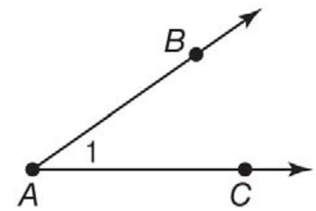
11. *M*(11, –2), *N*(–9, 13)

12. *E*(–2, 6), *F*(–9, 3)

13. *S*(10, –22), *T*(9, 10)

14. *K*(–11, 2), *L*(–19, 6)

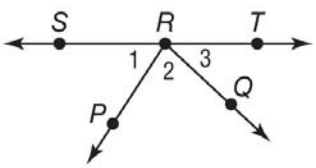
1-4 Angle Measure



Measure Angles If two non-collinear **rays** have a common endpoint, they form an **angle.** The rays are the **sides** of the angle. The common endpoint is the **vertex.** The angle at the right can be named as $\angle A$, $\angle BAC$, $\angle CAB$, or $\angle 1$.

A **right angle** is an angle whose measure is 90. An **acute angle** has measure less than 90. An **obtuse angle** has measure greater than 90 but less than 180.

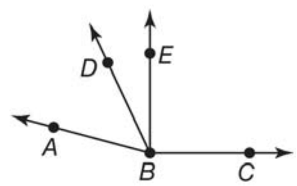
Example 1:



a. Name all angles that have *R* as a vertex.
Three angles are $\angle 1$, $\angle 2$, and $\angle 3$. For other angles, use three letters to name them: $\angle SRQ$, $\angle PRT$, and $\angle SRT$.

b. Name the sides of $\angle 1$.
$\overrightarrow{RS}$, $\overrightarrow{RP}$

Example 2: Classify each angle as *right, acute,* or *obtuse.* Then use a protractor to measure the angle to the nearest degree.



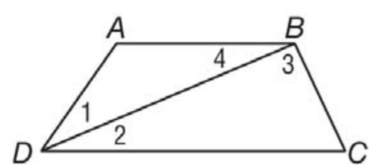
a. $\angle ABD$
Using a protractor, $m\angle ABD = 50$. $50 < 90$, so $\angle ABD$ is an acute angle.

b. $\angle DBC$
Using a protractor, $m\angle DBC = 115$. $180 > 115 > 90$, so $\angle DBC$ is an obtuse angle.

c. $\angle EBC$
Using a protractor, $m\angle EBC = 90$. $\angle EBC$ is a right angle.

Exercises

Refer to the figure at the right.

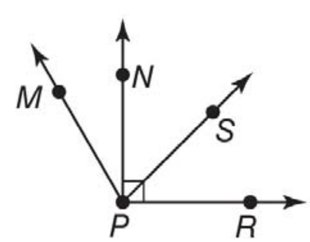


1. Name the vertex of $\angle 4$.

2. Name the sides of $\angle BDC$.

3. Write another name for $\angle DBC$.

Classify each angle as *right, acute,* or *obtuse.* Then use a protractor to measure the angle to the nearest degree.

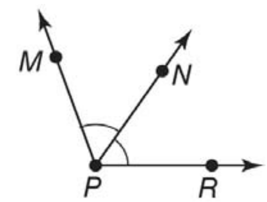


4. $\angle MPR$

5. $\angle RPN$

6. $\angle NPS$

1-4 Angle Measure (continued)



Congruent Angles Angles that have the same measure are **congruent angles.** A ray that divides an angle into two congruent angles is called an **angle bisector.** In the figure, $\overrightarrow{PN}$ is the angle bisector of ∠*MPR.* Point *N* lies in the interior of ∠*MPR* and ∠*MPN* ≅ ∠*NPR*.

Example: Refer to the figure above. If *m*∠*MPN* = 2*x* + 14 and *m*∠*NPR* = *x* + 34, find *x* and find *m*∠*NPR*.

Since $\overrightarrow{PN}$ bisects ∠*MPR,* ∠*MPN* ≅ ∠*NPR,* or *m*∠*MPN* = *m*∠*NPR.*

$2x + 14 = x + 34$

$2x + 14 - x = x + 34 - x$

$x + 14 = 34$

$x + 14 - 14 = 34 - 14$

$x = 20$

$m\angle NPR = 2x + 14$

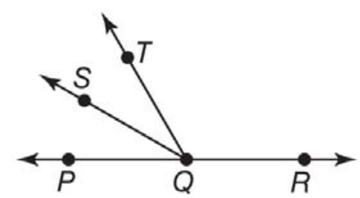
$= 2(20) + 14$

$= 40 + 14$

$= 54$

Exercises

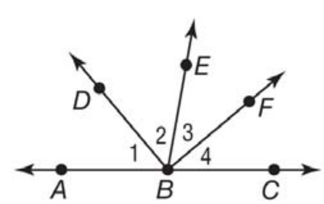
ALGEBRA In the figure $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are opposite rays. $\overrightarrow{QS}$ bisects ∠*PQT*.



1. If *m*∠*PQT* = 60 and *m*∠*PQS* = 4*x* + 14, find the value of *x*.

2. If *m*∠*PQS* = 3*x* + 13 and *m*∠*SQT* = 6*x* – 2, find *m*∠*PQT.*

ALGEBRA In the figure $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. $\overrightarrow{BF}$ bisects ∠*CBE*.



3. If *m*∠*EBF* = 6*x* + 4 and *m*∠*CBF* = 7*x* – 2, find *m*∠*EBF*.

4. If *m*∠3 = 4*x* + 10 and *m*∠4 = 5*x*, find *m*∠4.

5. If *m*∠3 = 6*y* + 2 and *m*∠4 = 8*y* – 14, find *m*∠*CBE.*

6. Let *m*∠1 = *m*∠2. If *m*∠*ABE* = 100 and *m*∠*ABD* = 2(*r* + 5), find *r* and *m*∠*DBE.*

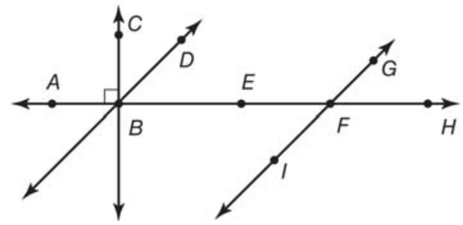
1-5 Angle Relationships

Pairs of Angles Adjacent angles are two angles that lie in the same plane and have a common vertex and a common side, but no common interior points. A pair of adjacent angles with non-common sides that are opposite rays is called a **linear pair**. **Vertical angles** are two nonadjacent angles formed by two intersecting lines.

Example: Name an angle or angle pair that satisfies each condition.

a. Two vertical angles

$\angle EFI$ and $\angle GFH$ are nonadjacent angles formed by two intersecting lines. They are vertical angles.



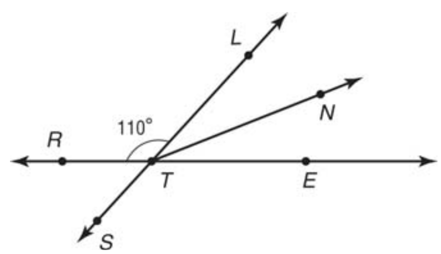
b. Two adjacent angles
$\angle ABD$ and $\angle DBE$ have a common vertex and a common side but no common interior points. They are adjacent angles.

c. Two supplementary angles
$\angle EFG$ and $\angle GFH$ form a linear pair. The angles are supplementary.

d. Two complementary angles
$m\angle CBD + m\angle DBE = 90$. These angles are complementary.

Exercises

Name an angle or angle pair that satisfies each condition.



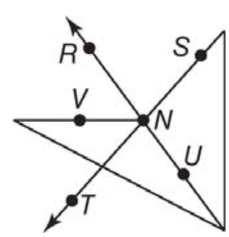
1. Two adjacent angles

2. Two acute vertical angles

3. Two supplementary adjacent angles

4. An angle supplementary to $\angle RTS$

For Exercises 5-7, use the figure at the right.



5. Identify two obtuse vertical angles.

6. Identify two acute adjacent angles.

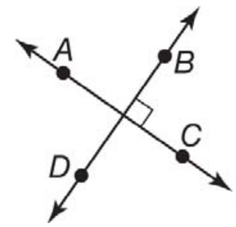
7. Identify an angle supplementary to $\angle TNU$.

8. Find the measures of two complementary angles if the difference in their measures is 18.

NAME ______________________________________ DATE ________________________ PERIOD __________

1-5 Angle Relationships (continued)

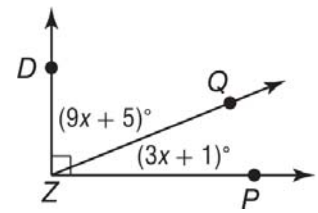
Perpendicular Lines: Lines, rays, and segments that form four right angles are **perpendicular.** The right angle symbol indicates that the lines are perpendicular. In the figure at the right, $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{BD}$, or $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$.



Example: Find *x* so that $\overrightarrow{DZ}$ and $\overrightarrow{ZP}$ are perpendicular

If $\overrightarrow{DZ} \perp \overrightarrow{ZP}$, then $m\angle DZP = 90$.

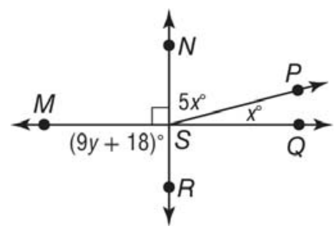
$m\angle DZQ + m\angle QZP = m\angle DZP$	Sum of parts = whole
$(9x + 5) + (3x + 1) = 90$	Substitution
$12x + 6 = 90$	Combine like terms.
$12x = 84$	Subtract 6 from each side.
$x = 7$	Divide each side by 12.



Exercises

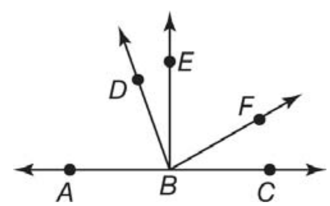
1. Find the value of *x* and *y* so that $\overleftrightarrow{NR} \perp \overleftrightarrow{MQ}$.

2. Find $m\angle MSN$.

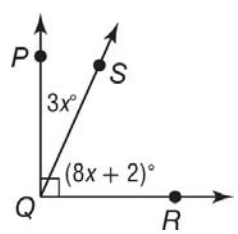


3. $m\angle EBF = 3x + 10$, $m\angle DBE = x$, and $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the value of *x*.

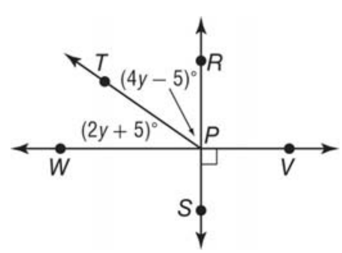
4. If $m\angle EBF = 7y - 3$ and $m\angle FBC = 3y + 3$, find the value of *y* so that $\overrightarrow{BE} \perp \overrightarrow{BC}$.



5. Find the value of *x*, $m\angle PQS$, and $m\angle SQR$.



6. Find the value of *y*, $m\angle RPT$, and $m\angle TPW$.



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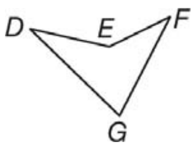
1-6 Two-Dimensional Figures

Polygons A **polygon** is a closed figure formed by a finite number of coplanar segments called **sides.** The sides have a common endpoint, are non-collinear, and each side intersects exactly two other sides, but only at their endpoints. In general, a polygon is classified by its number of sides. The vertex of each angle is a **vertex of the polygon.** A polygon is named by the letters of its vertices, written in order of consecutive vertices. Polygons can be **concave** or **convex.** A convex polygon that is both **equilateral** (or has all sides congruent) and **equiangular** (or all angles congruent) is called a regular polygon.

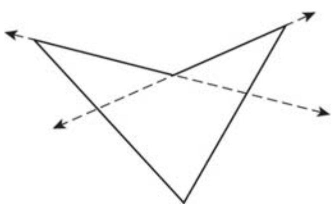
Example: Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular.*

a.



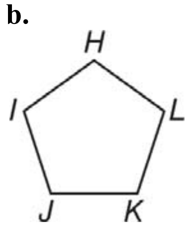
The polygon has four sides, so it is a quadrilateral.

Two of the lines containing the sides of the polygon will pass through the interior of the quadrilateral, so it is concave.



Only convex polygons can be regular, so this is an irregular quadrilateral.

b.



The polygon has five sides, so it is a pentagon.

No line containing any of the sides will pass through the interior of the pentagon, so it is convex.

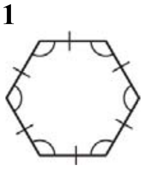
All of the sides are congruent, so it is equilateral. All of the angles are congruent, so it is equiangular.

Since the polygon is convex, equilateral, and equiangular, it is regular. So this is a regular pentagon.

Exercises

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

1



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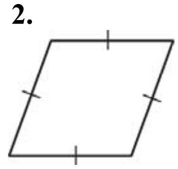

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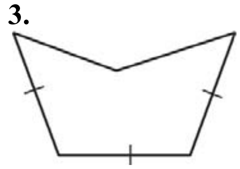

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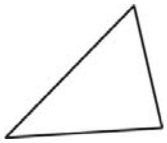

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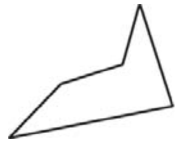

6.

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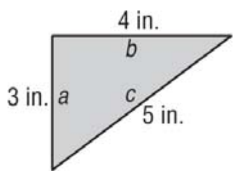
1-6 Two-Dimensional Figures (continued)

Perimeter, Circumference, and Area The **perimeter** of a polygon is the sum of the lengths of all the sides of the polygon. The **circumference** of a circle is the distance around the circle. The **area** of a figure is the number of square units needed to cover a surface.

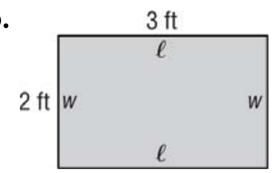
Example: Write an expression or formula for the perimeter and area of each. Find the perimeter and area. Round to the nearest tenth.

a.



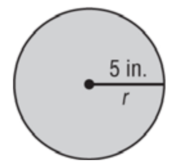
$P = a + b + c$
$= 3 + 4 + 5$
$= 12$ in.
$A = \frac{1}{2}bh$
$= \frac{1}{2}(4)(3)$
$= 6 \text{ in}^2$

b.



$P = 2\ell + 2w$
$= 2(3) + 2(2)$
$= 10$ ft
$A = lw$
$= (3)(2)$
$= 6 \text{ ft}^2$

c.

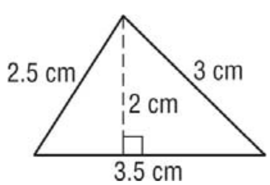


$C = 2\pi r$
$= 2\pi(5)$
$= 10\pi$ or about 31.4 in.
$A = \pi r^2$
$= \pi(5)^2$
$= 25\pi$ or about 78.5 in^2

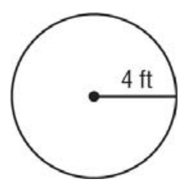
Exercises

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

1.



2.



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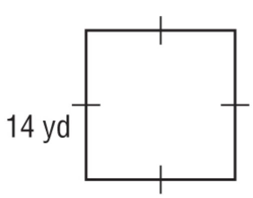

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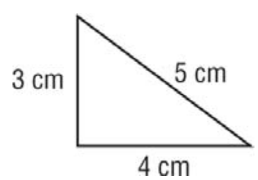
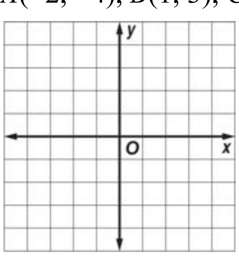
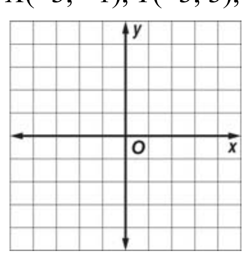


COORDINATE GEOMETRY Graph each figure with the given vertices and identify the figure. Then find the perimeter and area of the figure.

5. $A(-2, -4)$, $B(1, 3)$, $C(4, -4)$



6. $X(-3, -1)$, $Y(-3, 3)$, $Z(4, -1)$, $P(4, 2)$



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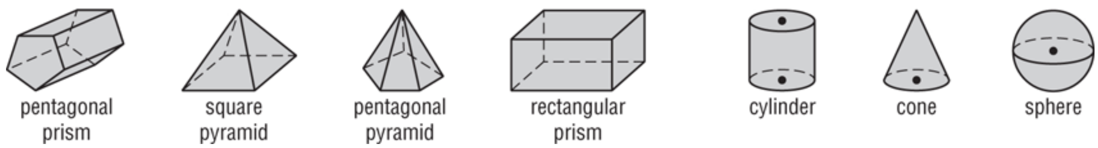
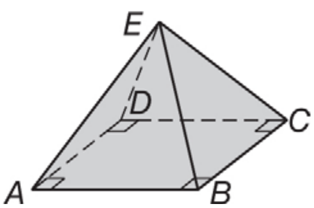
1-7 Three-Dimensional Figures

Identify Three-Dimensional Figures A solid with all flat surfaces that enclose a single region of space is called a **polyhedron.** Each flat surface, or **face,** is a polygon. The line segments where the faces intersect are called **edges.** The point where three or more edges meet is called a **vertex.** Polyhedrons can be classified as **prisms** or **pyramids.** A prism has two congruent faces called **bases** connected by parallelogram faces. A pyramid has a polygonal base and three or more triangular faces that meet at a common vertex. Polyhedrons or **polyhedra** are named by the shape of their bases. Other solids are a **cylinder**, which has parallel circular bases connected by a curved surface, a **cone** which has a circular base connected by a curved surface to a single vertex, or a **sphere**.



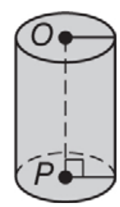
Example: Determine whether each solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the faces, edges, and vertices.

a.



The figure is a rectangular pyramid. The base is rectangle *ABCD*, and the four faces $\triangle ABE$, $\triangle BCE$, $\triangle CDE$, and $\triangle ADE$ meet at vertex *E*. The edges are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, and $\overline{DE}$. The vertices are *A, B, C, D,* and *E*.

b.

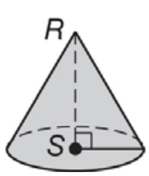


This solid is a cylinder. The two bases are $\odot O$ and $\odot P$. The solid has a curved surface, so it is not a polyhedron. It has two congruent circular bases, so it is a cylinder

Exercises

Determine whether each solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the faces, edges, and vertices.

1.



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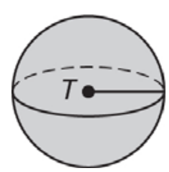

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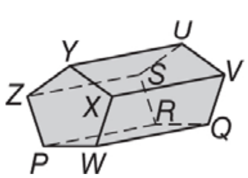

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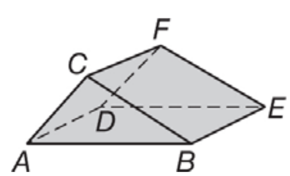

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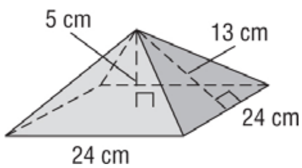
1-7 Three-Dimensional Figures (continued)

SURFACE AREA AND VOLUME Surface area is the sum of the areas of each face of a solid. Volume is the measure of the amount of space the solid encloses.

Example:

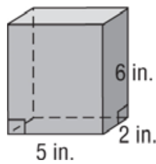
Write an expression or formula for the surface area and volume of each solid. Find the surface area and volume. Round to the nearest tenth.

a.



$S = \frac{1}{2}Pl + B$
$= \frac{1}{2}(96)(13) + 576$
$= 1200 \text{ cm}^2$
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(576)(5)$
$= 960 \text{ cm}^2$

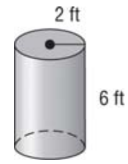
b.



$S = Ph + 2B$
$= (14)(6) + 2(10)$
$= 104 \text{ in}^2$

$V = Bh$
$= (10)(6)$
$= 60 \text{ in}^3$

c.



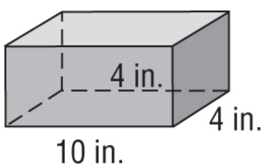
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(2)(6) + 2\pi(2)^2$
$= 32\pi$ or about 100.5 ft^2

$V = \pi r^2 h$
$= \pi(2)^2(6)$
$= 24\pi$ or about 75.4 ft^3

Exercises

Find the surface area of each solid to the nearest tenth.

1.



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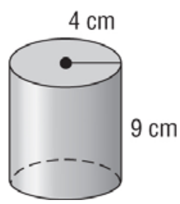

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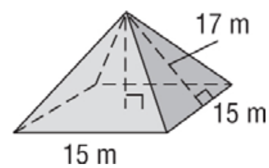

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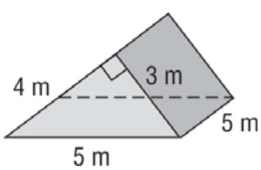

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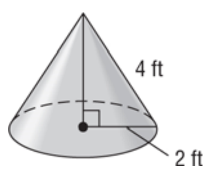

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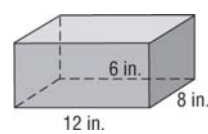
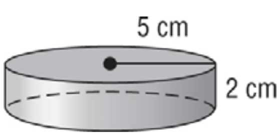


Find the volume of each solid to the nearest tenth.

7.



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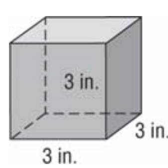

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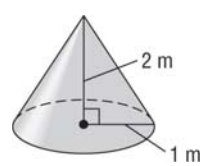

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