AP Calculus BC Summer Assignment

Due Date: First Day of School

Name__________________

Welcome to AP Calculus BC!

This is an exciting, challenging, fast paced course that is taught at the college level. We have a lot of material to cover before the AP exam in May and, as such, this mandatory packet is designed to review much of that material for you.

Spend time working on this over the summer. If you struggle with specific areas, use your old notes, previous textbook(s), and/or the internet as a resource. Do not wait until the last minute to complete this assignment. Most of the packet should be completed without a calculator, although there are few sections that indicate calculator use is expected. A couple of items that you should be aware of:

1. Know your trig!! You should know your unit circle COLD. Trigonometry will often show up in the middle of a problem and the faster you are with your facts, the better equipped you will be to solve problems. If you want to practice filling out a unit circle (maybe a few times throughout the summer) go to **Embedded Math** (http://www.embeddedmath.com/) and click on worksheets. You can find a blank unit circle to print. This is important!

2. Know your basics from exponential and logarithmic functions (know that $\ln e = 1$, $\ln 1 = 0$, and that *log* has an understood base of 10, etc.).

3. Back to Trig!!! Know your double angle identities for sine and cosine. Know your (3) trig Pythagorean identities, and know your reciprocal identities.

The greatest skill you can have to be successful in this course is to be a consistent worker. Being good at math will only take you so far. Being willing to work will get you to the finish line! You need to complete homework when it is assigned, pay attention in class, and meet with me outside of class as needed. I encourage you to form a small study group to work on homework and to prepare for assessments.

I look forward to a successful year of AP Calculus!

Section 1: Limits and Continuity

In questions 1-5, find the limit:

1. $\lim_{x\to 4} \frac{\sqrt{x-3}-1}{x-4}$

2. $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

3. $\lim_{x\to 0} \frac{1-\cos x}{\sin x}$

4. $\lim_{x\to \frac{\pi}{4}} \frac{4x}{\tan x}$

5. $\lim_{x\to 0} \frac{\cos x-1}{x}$

In questions 6-9, find the limit (if it exists.) If it does not exist, explain why.

6. $\lim_{x\to -3^+} \frac{1}{x+3}$

7. $\lim_{x\to 6^-} \frac{x-6}{x^2-36}$

8. $\lim_{x\to 4^-} \frac{\sqrt{x}-2}{x-4}$

9. $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^3+1, & x<1 \\ \frac{1}{2}(x+1), & x \geq 1 \end{cases}$

In questions 10-13, find the vertical asymptotes (if any) of the graph of the function.

10. $f(x) = \frac{5}{(x-2)^4}$

11. $f(x) = \frac{6x}{36-x^2}$

12. $f(x) = \frac{2x+1}{x^2-64}$

13. $f(x) = \csc \pi x$

In questions 14-18, find the one-sided limit (if it exists.)

14. $\lim_{x \to 1^-} \frac{x^2+2x+1}{x-1}$

15. $\lim_{x \to \frac{1}{2}^+} \frac{x}{2x-1}$

16. $\lim_{x \to 0^+} \frac{\sin 4x}{5x}$

17. $\lim_{x \to 0^+} \frac{\sec x}{x}$

18. $\lim_{x \to 0^+} \frac{\csc 2x}{x}$

Section 2: Differentiation

In questions 19-37, find the derivative of the function:

19. $f(x) = x^3 - 11x^2$

20. $f(x) = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

21. $h(x) = \frac{8}{5x^4}$

22. $h(x) = \sqrt{x} \sin x$

23. $f(t) = 2t^5 \cos t$

24. $y = \frac{x^4}{\cos x}$

25. $y = 2x - x^2 \tan x$

26. $g(x) = 3x \sin x + x^2 \cos x$

27. $y = (7x+3)^4$

28. $f(x) = \frac{1}{(5x+1)^2}$

29. $y = 5\cos(9x+1)$

30. $f(x) = \frac{3x}{\sqrt{x^2+1}}$

31. $h(x) = \left(\frac{x+5}{x^2+3}\right)^2$

32. $g(x) = \ln\sqrt{2x}$

33. $y = \ln\left(\frac{4x}{x-6}\right)$

34. $g(t) = t^2 e^t$

35. $y = \sqrt{e^{2x} + e^{-2x}}$

36. $y = 3e^{\frac{-3}{t}}$

37. $f(x) = x(4^{-3x})$

For question 38, use the position function $s(t) = -16t^2 + v_\circ t + s_\circ$ for free-falling objects:

38. A ball is thrown straight down from the top of a 600-foot building with an initial velocity of -30 feet per second.
    a. Determine the position and velocity functions for the ball.
    b. Determine the average velocity on the interval [1,3].
    c. Find the instantaneous velocities when $t = 1$ and $t = 3$.
    d. Find the time required for the ball to reach ground level.
    e. Find the velocity of the ball at impact.

In questions 39-40, find the second derivative of the function:

39. $f(x) = \cot x$

40. $y = (8x + 5)^3$

In questions 41-44, find and evaluate the derivative of the function at the given point:

41. $f(x) = \sqrt{1 - x^3}$, $(-2, 3)$

42. $f(x) = \frac{4}{x^2+1}$, $(-1, 2)$

43. $y = \frac{1}{2}\csc 2x$, $(\frac{\pi}{4}, \frac{1}{2})$

44. $y = \csc 3x + \cot 3x$, $(\frac{\pi}{6}, 1)$

In questions 45-49, find $\frac{dy}{dx}$ by implicit differentiation.

45. $x^2 + y^2 = 64$

46. $x^3y - xy^3 = 4$

47. $\cos(x + y) = x$

48. $y \ln x + y^2 = 0$

49. $\cos x^2 = xe^y$

50. A point moves along the curve $y = \sqrt{x}$ in such a way that the y-value is increasing at a rate of 2 units per second. At what rate is x changing for each of the following values?
    a. $x = \frac{1}{2}$
    b. $x = 1$
    c. $x = 4$

51. All edges of a cube are expanding at a rate of 8 centimeters per second. How fast is the surface area changing when each edge is 6.5 centimeters?

Section 3: Applications of Differentiation

In questions 52-55, find the absolute extrema of the function on the closed interval:

52. $f(x) = x^2 + 5x$, $[-4, 0]$

53. $f(x) = \frac{4x}{x^2+9}, [0\,,4]$

54. $g(x) = 2x + 5\cos x,\ \ [0\,,2\pi]$

55. $f(x) = \sin 2x, [0\,,2\pi]$

56. Can the Mean Value Theorem be applied to the function $f(x) = \frac{1}{x^2}$ on the interval $[-2, 1]$? Explain.

In questions 57-61, identify the open intervals on which the function is increasing or decreasing:

57. $f(x) = x^2 + 3x - 12$

58. $f(x) = (x-1)^2(x-3)$

59. $f(x) = \sin x + \cos x, [0\,,2\pi]$

60. $f(x) = \frac{x^3-8x}{4}$

61. $g(x) = \frac{3}{2}\sin\left(\frac{\pi x}{2} - 1\right), [0\,,4]$

In questions 62-64, find the points of inflection and discuss the concavity of the graph of the function:

62. $g(x) = x\sqrt{x+5}$

63. $f(x) = x + \cos x\,, [0\,,2\pi]$

64. $f(x) = \tan\frac{x}{4}, (0, 2\pi)$

In questions 65-69, find the limit:

65. $\lim_{x\to\infty}\left(8 + \frac{1}{x}\right)$

66. $\lim_{x\to-\infty}\frac{3x^2}{x+5}$

67. $\lim_{x \to -\infty} \frac{\sqrt{x^2+x}}{-2x}$

68. $\lim_{x \to \infty} \frac{5 \cos x}{x}$

69. $\lim_{x \to -\infty} \frac{x}{2 \sin x}$

Section 4: Integration

In questions 70-80, find the indefinite integral:

70. $\int \frac{x^4+8}{x^3}\, dx$

71. $\int (2x - 9 \sin x)\, dx$

72. $\int \frac{x^2}{\sqrt{x^3+3}}\, dx$

73. $\int \frac{x+4}{x^2+8x-7}\, dx$

74. $\int \sin^3 x \cos x\, dx$

75. $\int \frac{\sin x}{\sqrt{\cos x}}\, dx$

76. $\int \sec 2x \tan 2x\, dx$

77. $\int \frac{1}{7x-2}\, dx$

78. $\int \frac{\sin x}{1+\cos x}\, dx$

79. $\int x^2 e^{x^3+1} dx$

80. $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\, dx$

81. A ball is thrown vertically upward from ground level with an initial velocity of 96 feet per second. Use $a(t) = -32$ feet per second per second as the acceleration due to gravity. (Neglect air resistance.)

a. How long will it take the ball to rise to its maximum height? What is the maximum height?
b. After how many seconds is the velocity of the ball one-half the initial velocity?
c. What is the height of the ball when its velocity is one-half the initial velocity?

In questions 82-92, evaluate the definite integrals:

82. $\int_2^3 (t^2 - 1)\, dt$

83. $\int_4^9 x\sqrt{x}\, dx$

84. $\int_1^4 \left(\frac{1}{x^3} + x\right) dx$

85. $\int_0^{\frac{3\pi}{4}} \sin\theta\, d\theta$

86. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 t\, dt$

87. $\int_0^3 \frac{1}{\sqrt{1+x}}\, dx$

88. $\int_0^\pi \cos\frac{x}{2}\, dx$

89. $\int_1^e \frac{\ln x}{x}\, dx$

90. $\int_0^\pi \tan\frac{\theta}{3}\, d\theta$

91. $\int_{\frac{1}{2}}^2 \frac{e^{\frac{1}{x}}}{x^2}\, dx$

92. $\int_1^3 \frac{e^x}{e^x - 1}\, dx$

In questions 93-94, find the average value of the function over the given interval:

93. $f(x) = \frac{1}{\sqrt{x}}, [4\,, 9]$

94. $f(x) = x^3, [0\,, 2]$

In questions 95-96, find the particular solution of the differential equation that satisfies the initial condition:

95. $y' \cos^2 x + y - 1 = 0\,, y(0) = 5$

96. $y' + y \tan x = \sec x + \cos x, y(0) = 1$

Section 5: Applications of Integration

In questions 97-99, find the area of the region bounded by the graphs of the equations:

97. $y = 6 - \frac{1}{2}x^2, y = \frac{3}{4}x, x = -2, x = 2$

98. $x = y^2 - 2y, x = -1, y = 0$

99. $y = \sin x, y = \cos x, \frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$

In questions 100-101, find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given line(s):

100. $y = \sqrt{x}, y = 2, x = 0$
   a. the x-axis
   b. the y-axis
   c. the line $y = 2$
   d. the line $x = -1$

101. $y = e^{-x}, y = 0, x = 1, x = 1$ revolved about the x-axis

Section 6: Partial fraction decomposition

Find the partial fraction decomposition of:

102. $\dfrac{5x+11}{3x^2-5x-2}$

103. $\dfrac{x^2+2}{(x-1)(x+2)(x-3)}$

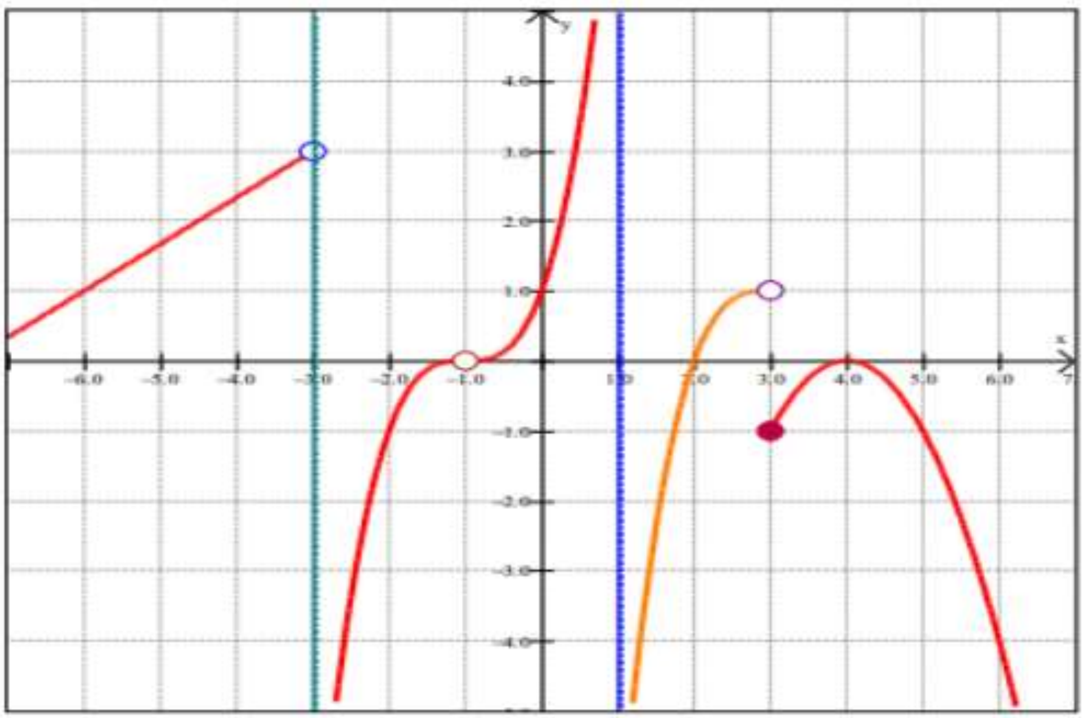
104. $\dfrac{x^4+2x+7}{x^2+3x+2}$ *(hint: long division first…)*

Section 7: More limits and continuity

105. Consider the function $f(x) = \begin{cases} x^2 + kx & x \le 5 \\ 5\sin\left(\frac{\pi}{2}x\right) & x > 5 \end{cases}$ In order for the function to be continuous at x = 5, what must the value of k be?

106.

Determine if the following limits exist based on the graph below of $p(x)$. If the limit exists, state the value. Note that x = -3 and and x =1 are vertical asymptotes.



a) $\lim_{x\to 1^+} p(x)$

b) $\lim_{x\to 1^-} p(x)$

c) $\lim_{x\to 1} p(x)$

d) $\lim_{x\to 3^-} p(x)$

e) $\lim_{x\to 3^+} p(x)$

f) $\lim_{x\to 3} p(x)$

g) $\lim_{x\to -1^+} p(x)$

h) $\lim_{x\to -1^-} p(x)$

i) $\lim_{x\to -1} p(x)$

j) $\lim_{x\to 2^+} p(x)$

k) $\lim_{x\to 2^-} p(x)$

l) $\lim_{x\to 2} p(x)$

107. Find the limit (if it exists). If does not exist, explain. Support with the graph.

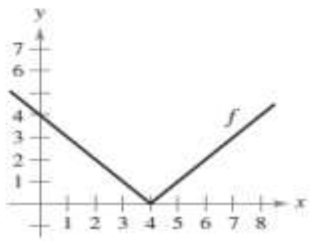
$$\lim_{x \to 4^-} (5[\![x]\!] - 7)$$

108. Use the Intermediate Value Theorem to show that $f(x) = 2x^3 - 3$ has a zero in the interval $[1, 2]$.

Section 8: More derivatives

Sketch the graph of f'. Explain how you got the answer.

109.



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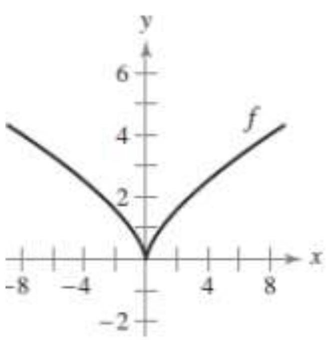

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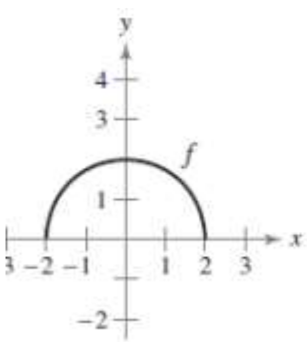

112. Sketch a graph of a function whose derivative is always negative. Explain how you found your answer.

Determine whether the statement is true or false. If false, explain why or show the example that shows it is false.

113. If a function is continuous at a point, then it is differentiable at that point.

114. If a function has derivatives from both the right and the left at a point, then it is differentiable at that point.

115. If a function is differentiable at a point, then it is continuous at that point.

, **use the given information to find $f'(2)$.**

$g(2) = 3$ **and** $g'(2) = -2$

$h(2) = -1$ **and** $h'(2) = 4$

116. $f(x) = 2g(x) + h(x)$

117. $f(x) = g(x)h(x)$

118. Find the equation of the line tangent and normal to the graph of $3xy^3 - 4x^2 + 2y = -25$ at the point (3, 1)

119. The lines tangent to the graphs of $g(x)$ and $h(x)$ are perpendicular to one another when x = 2. If $g(x) = \ln(2x^3 + 5)$, find the slope for h(x) at x = 2.

120. Find the equation of the line tangent to $f(x) = 2x(3x^2 - 4)^2$ at x = 1.

121. Given $= \frac{5e^{3x}\sqrt{2x-5}}{\sin^2(2x)}$, fine dy/dx. Hint…do NOT use quotient rule. Take the log or natural log of both sides of equation. Expand the right hand side and then take the derivative!!!

122. Given $y = x^{\tan x}$, find dy/dx. Again…take log or natural log of both sides first (to bring down exponent) and then take the derivative.

123. Given $f(x) = \sqrt{x+1}$.

a) Find $f'(3)$ and write the equation of the tangent line and normal line when x = 3. Provide a quick (but accurate) sketch of the curve and the tangent and normal lines.

b) Find $f'(3)$ using the limit definition of the derivative.

c) Find $f'(3)$ using the alternate definition of the derivative.

Section 9: Related rates

124: ***Area*** The radius r of a circle is increasing at a rate of 4 centimeters per minute. Find the rates of change of the area when (a) $r = 8$ centimeters and (b) $r = 32$ centimeters.

125. ***Volume*** The radius r of a sphere is increasing at a rate of 3 inches per minute.

(a) Find the rates of change of the volume when $r = 9$ inches and $r = 36$ inches.

(b) Explain why the rate of change of the volume of the sphere is not constant even though dr/dt is constant.

126. ***Volume*** At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?

127. ***Shadow Length*** A man 6 feet tall walks at a rate of 5 feet per second away from a light that is 15 feet above the ground (see figure). When he is 10 feet from the base of the light,

(a) at what rate is the tip of his shadow moving?

(b) at what rate is the length of his shadow changing?