POSNACK SCHOOL

Summer Assignment 2021 - 2022・AP Physics C: Mechanics

Course: Advanced Placement Physics C: Mechanics

Textbook: Randall Knight. *Physics for Scientists and Engineers: A Strategic Approach w/ Modern Physics* 4e

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Welcome to AP Physics C: Mechanics! Physics C: Mechanics is a unique course that explores the concepts of Physics through the lens of calculus and allows students to model and solve real-world problems with a wide variety of methodologies.

Physics C: Mechanics, like other AP Courses offered at Posnack School, is designed to be rigorous, fast-paced and challenging and is taught at the college-level. As such, there is a lot of material to cover in a relatively short amount of time leading into the AP Exam. This summer assignment is designed to review and bolster your understanding of key concepts of Physics 1 and Calculus AB that will best enable you to succeed in this course. Please complete this assignment by the end of the first week in class and bring questions on Day 1. Be prepared to take an assessment on the material early in the school year. While group study is encouraged, cheating will not be tolerated.

AP Physics C: Mechanics will be a challenging course at points, but you are up to the task and will have every opportunity to be successful! I am excited to get to know and to work with all of you in the upcoming school year!

<u>Review of Calculus AB Topics</u>

There are many great online resources to review the concepts of AP Calculus AB that will be most utilized in the AP Physics C: Mechanics course. One such resource is a series of videos made available by the University of Houston at the following website: click here.

I recommend reviewing videos 1,2,5,6, 8 and 10 which relate to Limits, Derivatives and Integrals and using knowledge from those videos to answer the following set of questions.

<u>Derivatives:</u>

1. Explain in your own words the meaning of taking the derivative of a function.
2. Explain in your own words the meaning of taking the second derivative of a function.
3. If f(x) is a continuous and differentiable function, explain in words a calculus procedure to find all of the local minima and maxima of that function using only the first derivative. Where are all of the locations that a max/min value can exist?
4. How can the second derivate of a function help to distinguish maxima from minima?
5. Find the **first** and **second** derivatives of the following functions:

a. $f(x) = x^2 + 3x + 2$
b. $g(x) = -3x^3 - 2x + 4$
c. $h(x) = -x^4 + 4x^2 + 2x$
d. $f(x) = 2x^5 + 4x^2 + 2x$
e. $g(x) = -2x^{10} + x^2 - x$
f. $h(x) = x^4 + 4x^3 + 2x - 4$
g. $f(x) = \sin(x^2)$
h. $g(x) = \sin^2(x)$

6. For the following items, use the chain rule, product rule and quotient rules as appropriate to find the **first** derivative of the function. Please do not multiply out polynomials to solve.
   a. $f(x) = (4x^2 + 2x)^3$
   b. $g(x) = \frac{3-x^3}{x^2-1}$
7. For the following functions, find local minima and local maxima and determine which are minima and which are maxima. Where appropriate, use the chain rule, product rule or quotient rule to evaluate derivatives.
   a. $f(x) = x^3 - 2x^2 + 3x + 1$
   b. $g(x) = \frac{(x-3)^2}{2x^3}$
   c. $h(x) = (2x^2+1)^2$

Integrals

1. Find the integral (antiderivative) of each of the following items. Do not forget to add constants where necessary!
   a. $\int 3x^2 + 2x + 1\, dx$
   b. $\int 5x - x^5 + 8\, dx$
   c. $\int \sqrt{x} + x^{2/3} + \frac{x^2}{x^{1/3}}$
   d. $\int (sinx - cosx)dx$
   e. $\int \frac{1}{x} dx$

Quick Word Problem!

A ball is thrown vertically into the air from an initial height of 10 meters (10m) at an initial velocity of 3 meters per second (3 m/s). the formula that represents the ball's height (displacement) relative to the floor at time t is given by $h(t) = -4.9t^2 + 3t + 10$. Hint: Velocity is the first derivative of the displacement function and acceleration is the second derivative of the displacement function.

- What is the instantaneous velocity of the ball at the moment it hits the ground? You may be interested to solve for the time at which distance to the floor is equal to 0.
- At what time does the ball reach its maximum height?
- What is the acceleration of the ball 0.3 seconds after it is first thrown?
- Does the acceleration of the ball ever change?

Review of Physics Topics

As with Calculus, there are many great resources available online that review key concepts from AP Physics 1. The following link provides one such resource that is easily searchable: click here.

In this assignment, we will brush up on concepts related to Motion, Vectors, and Forces.

Topic 1: Motion in One Dimension

In analyzing objects in motion, there are four basic quantities that we want to keep track of. These quantities are time, displacement, velocity and acceleration. Time is a scalar meaning that it has a magnitude but no directional component, whereas the other three are vectors with both magnitude and direction.

Displacement:

Displacement represents the net distance traveled by an object. Displacement is a vector, so it also has a direction associated with it. If you start in a particular location and walk 10m east, your displacement from your starting position is 10m east. If you then walk 5m towards the west, although you have traveled a total distance of 15m, your net displacement is only 5m east relative to your starting position. Displacement is the net difference between your final position x_f and your initial position x_0.

Speed and Velocity:

Velocity is a measure of displacement over time with an associated direction, hence it is a vector. Speed is a scalar quantity that references how fast an object is moving. Imagine you are leaving home to head to school and are walking east at 4m/s towards campus. You realize that you left your bag at home after 1 minute and decide to sprint back home at 8m/s west. Your sprint speed is twice as fast as your walking speed, so you make it home in half the time (30s) for a total roundtrip time of 90 seconds. Your average speed can be calculated by taking the total distance traveled of 480m (240m east + 240m west) and dividing it by the time elapsed of 90 seconds = 5.333 m/s. However, because your total net displacement in that roundtrip is 0m (you started exactly where you ended), your average velocity is 0m/s.

Acceleration:

Acceleration is a measure of the rate of change in an object's velocity. Its value can be negative, zero or positive. Imagine you are driving a car. When you first start the ignition, you are parked and not moving and your initial velecoity, v_0, is 0 mph. Once you eventually get onto the highway, you are now traveling at a final velocity, v_f, of 60 mph. As your velocity has grown over that period of time, there must have been an event of positive acceleration! In periods in which the car is moving at a constant velocity (think about being in cruise control on the highway), acceleration is 0. When you apply the brakes and move from a higher speed to a lower speed, acceleration is negative!

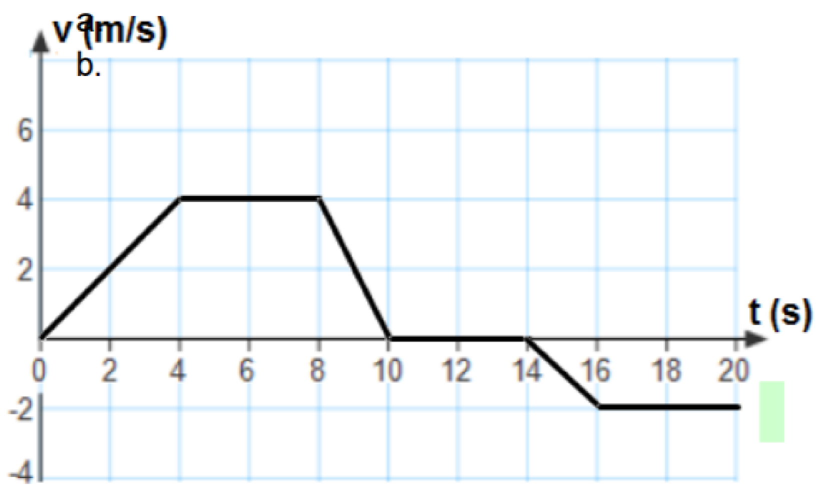
Kinematic Equations

There are 4 kinematic equations that describe the motion of an object. You can use these equations to simply determine an object's location, total displacement, velocity and the times associated for the objects to reach certain speeds or travel certain distances. These equations assume that motion is measured from t=0 and that acceleration is constant for the object.

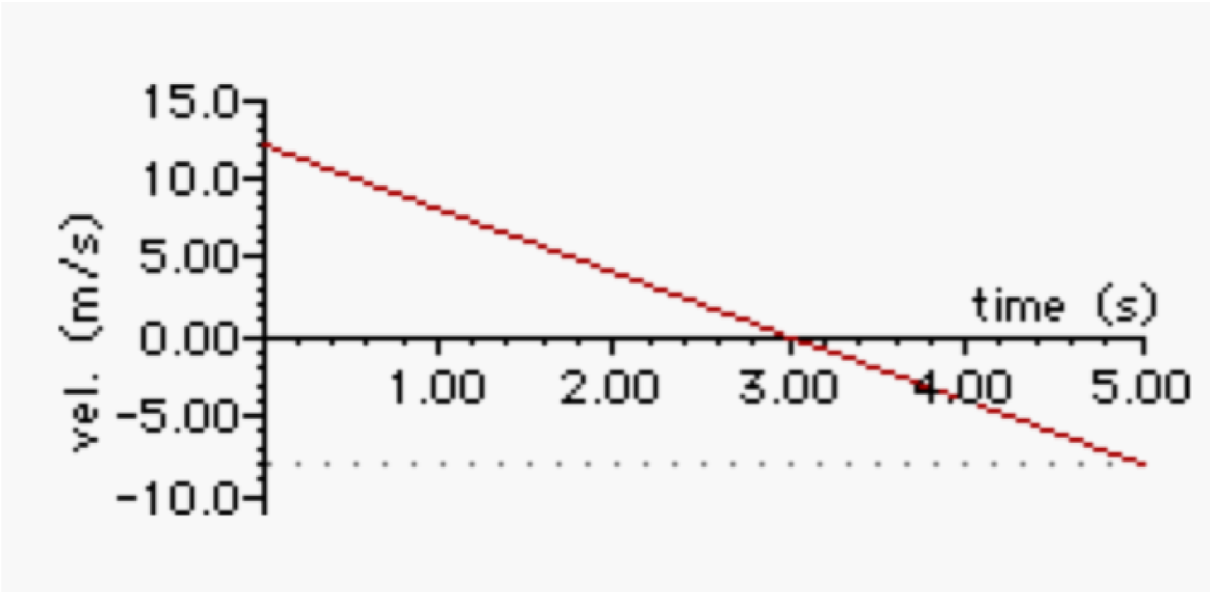
1. $\mathbf{V_f = V_0 + at}$ (final velocity = initial velocity + acceleration multiplied by elapsed time)
2. $\mathbf{X = V_0 t + \frac{1}{2}at^2}$
3. $\mathbf{X = \frac{v_f + v_0}{2} t}$ (total displacement is average velocity multiplied by time)
4. $\mathbf{V_f^2 = V_0^2 + 2ad}$ (where d = displacement)

The quantities of displacement, velocity and acceleration are all related via concepts of Calculus! If you were to graph an objects displacement relative to time, the graph's first derivative (slope) represents the objects instantaneous velocity and its second derivative (concavity) represents the object's instantaneous acceleration. Likewise, acceleration is the first derivative of the velocity graph relative to time and the area under the velocity graph represents net displacement.

Exercises



1. The graph above represents a walker's velocity over a 20 second interval. A positive value indicates that the walker is moving in the forward direction, whereas a negative value indicates that the walker turned around and is heading in the opposite direction. Evaluate the following including units:
    a. Indicate the time intervals in which the speed (magnitude of the velocity) is decreasing.
    b. Indicate the time intervals in which the walker is not moving.
    c. Determine the net displacement of the walker at t=4 if they are located at position X = 0 when t=0.
    d. Determine the total distance traveled by the walker over the first 10s of their journey.
    e. Determine the acceleration of the walker over the following intervals: [0s,4s], [4s,8s], [8s,10s], [10s,14s], [14s,16s] and [16s,20s]



2. After a long soccer practice, Jenny begins to walk home up a steep hill. She gives the soccer ball a kick up the hill and continues to walk towards the ball and catches up with it as it rolls back down the hill towards her. The graph above represents the ball's motion as it climbs the hill and eventually begins to roll back down. Evaluate the following:
    a. At what time did the ball change directions from rolling uphill to rolling downhill? How do you know?
    b. What is the acceleration of the ball as it rolls up the hill? What is the acceleration of the ball as it moves down the hill?
    c. How far up the hill does the ball travel before starting to roll down?
    d. What is the total distance that the ball travels both up and down the hill (not net displacement but total distance)?
3. You are driving on the road at 15.65 m/s and 50m before reaching a traffic light, you notice the light turn yellow. You accelerate your car at a constant rate for exactly 3 seconds and reach the intersection before the light turns red. What is your speed when you reach the traffic light?
4. A car travels up a 1000m hill at 25m/s and returns back down the hill at 40m/s (the upwards stretch is 1000m and the downward stretch of road is also 1000m for 2000m total road). What is the car's average speed over the duration of the hill and how many seconds elapsed?
5. An engineer is designing a runway for an airport. Of all of the planes that will use the airport, the place with the slowest acceleration accelerates at $3m/s^2$. If the plane is stopped at one end of the runway before accelerating at $3m/s^2$ and needs to reach a speed of 65 m/s to take off, what is the minimum length of runway needed to accommodate the airplane?
6. A baseball pitcher throws a baseball vertically at 40m/s. How high does the ball travel before eventually falling back down to the earth? Hint: Velocity at the top of the trajectory is 0m/s and the acceleration of the ball due to gravity is a constant $-9.8m/s^{2.}$
7. A sports car is involved in an accident. At the scene of the crash, police measured skid marks that were 290m long that were left by the car's tires as the driver decelerated at a constant rate to a stop. An unreliable witness claims that the driver took exactly 10 seconds to come to a stop after applying the brakes before admitting that it could have easily been either 9 or 11 seconds. What is the range of speeds that the driver might have been traveling prior to applying the brakes? Hint: first solve for the necessary acceleration that the car must have experienced to stop in 290m over 9 or 11 seconds before solving for velocity.

Topic 2: Scalars and Vectors

What is a vector?

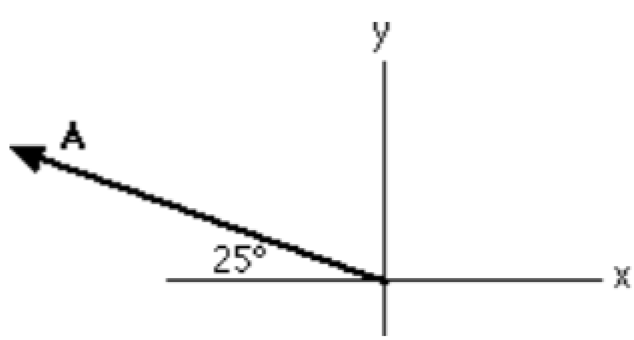
As we move on from analyzing motion in one dimension to analyzing motion in two or three dimensions, it is critical to distinguish between vectors and scalars. A scalar is a quantity that is just a number with a unit like mass (200kg) or temperature (70 degrees). A vector, on the other hand, has BOTH a magnitude and a direction, like velocity (20mph east). Velocity is a combination of a speed scalar with a corresponding direction. Examples of vectors in physics are displacement, velocity, acceleration and force.

A crucial difference with scalars and vectors is the meaning of plus and minus signs. A scalar with a plus sign can mean something very different than a scalar with a minus sign. Think of the differences between -50 degrees and +50 degrees Fahrenheit! With vectors, the sign simply tells you something about direction. Moving +20mph East is equivalent to moving -20mph West. Both have speeds of 20mph so the -20 isn't slower than the +20 despite -20 being a smaller number to +20. With a vector, the negative sign can always be negated and incorporated into the direction of the vector (why say -20mph west when you can say +20mph east?), but sometimes having the additional context is important for setting up and evaluating problems.

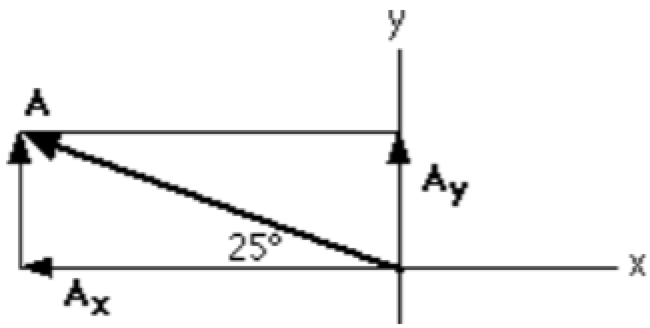
Vector Components

A vector pointing in a random direction in the X-Y Plane has X and Y components associated with it. It can be split into two distinct vectors, one in the X direction (X-component) and one in the Y direction (Y-component). Imagine the original vector as the hypotenuse of a right triangle and the components are the two legs in the X and Y direction. Added back together, the X and Y components give us the original vector. The easiest way to add and subtract vectors is to split the vectors into their respective components, add the X components and Y components separately and then recombine back into the new resultant vector.

Trigonometry is very useful in calculating X and Y components of vectors. I like to use the mnemonic device SOHCAHTOA to help with basic trigonometric evaluations. SOH-CAH-TOA represents that the SINE function is Opposite over Hypotenuse, COSINE is Adjacent over Hypotenuse and TANGENT is Opposite over Adjacent.



In the above diagram for vector A, we have drawn the Vector at 25 degrees above the Xaxis. Imagine vector A has a magnitude of 10meters. Given that we generally start counting degrees on the unit circle at 0 degrees from the right-most point and move in the counterclockwise direction, this vector is equivalent to 10 meters at 155 degrees (left x axis is 180 degrees and we are 25 clockwise to it so 180-25=155).



Drawing on the X and Y components of vector A shows that A_y and A_x are two legs of a right triangle with a hypotenuse of 10 meters drawn at 155 degrees. As SINE(Θ) = O/H and A_y = Opposite and Hypotenuse = 10m, we can algebraically re-arrange to get that A_y = 10sin(155deg) = 4.226m and likewise A_x = 10cos(155deg) = -9.063m (note that the value should carry the negative sign as the vector points in the negative X direction). Also note that using the Pythagorean Theorem, $A_y^2 + A_x^2 = 10^2$ and the two calculated values do indeed represent two legs of a right triangle relative to that hypotenuse.

Adding and Subtracting Vectors

When adding two vectors together, you will create a third vector that has its own magnitude and direction. Say vector C is the resultant of adding together vectors A + B. A simple way to perform the addition is with the following properties of vector addition:

$C_y = A_y + B_y$

$C_x = A_x + B_x$

The magnitude of C can be found using the Pythagorean Theorem:

Magnitude C = $\sqrt{C_x^2 + C_y^2}$

To calculate the direction of C, we can recognize that TANGENT = O/A where O= C_y and A = C_x. Then, tan(Θ) = C_y/C_x and $\tan^{-1}(C_y/C_x) = \Theta$.

Note that in the previous example, we calculated that the Y component = 4.226 and the X component = -9.063. C_y/C_x = -0.46629 and $\tan^{-1}(-0.46629)$ = -25 degrees which is equivalent to 360-25 = 335 degrees which is not equal to the 155 degrees stated in the original problem! When performing the division of C_y/C_x = -0.46629, the value of -0.46629 loses respect as to which of the components was negative! We need to use the context we have about which value is negative to correctly place the vector in the coordinate plane. 335 degrees and 155 degrees are exactly 180 degrees apart so they are mirrored across the original from one another depending on if X or Y is the negative quantity.

<u>Exercises</u>

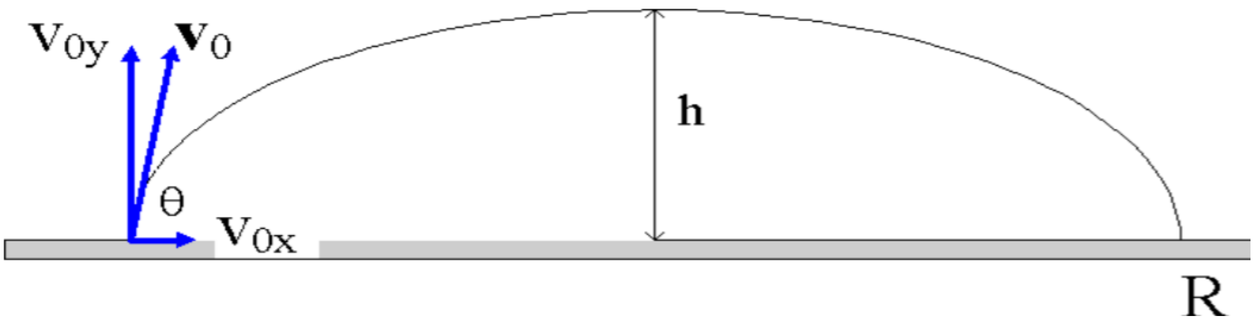
1. A bus makes three displacements in the following order: 1) 58 miles at 38 degrees east of north. 2) 69 miles at 46 degrees west of north 3) 75 miles 45 degrees south of east. Perform the following exericses:
    a. Draw a clear diagram showing all three displacement vectors with respect to a coordinate system.
    b. Find the X and Y components of Displacement 1 (D_1)
    c. Find the X and Y components of Displacement 2 (D_2)

d. Find the X and Y components of Displacement 3 (D_3)
e. Find the magnitude of the resultant vector $D_1 + D_2 + D_3$. What does this represent?
f. Find the direction of the resultant vector. What does this represent?

2. Which of the following statements are true of vectors and scalars? If the statement is false, state the corrected version of the statement.
   a. A vector quantity always has a direction associated with it
   b. A scalar quantity can have a direction associated with it.
   c. Vectors can be added together while scalar quantities cannot.
   d. Vectors can be represented by an arrow on a diagram. The length of the arrow represents the vector's magnitude and the direction it points represents the vector's direction.

Topic 3: Projectile Motion

A projectile is fired at an initial velocity of v_0 and an angle Θ above the horizon. How far will the projectile go? How high will it rise? These questions can be answered using vectors and the kinematic equations!



Denote the projectile's Range R and the maximum height of its trajectory as h. Let us create a coordinate system whose origin is at the launch point and whose X and Y axes are aligned with the X and Y components of the vector. Recall that originally, $V_y = V\sin(\Theta)$ $V_x = V\cos(\Theta)$. While the range of the projectile moves in two dimensions (X&Y), we can simplify this analysis by thinking about the X and Y motions independently of one another!

The Horizontal Component V_x

If we neglect the effects of air resistance as the projectile flies through the air, we can say that the velocity of the projectile in the X-direction is constant. That means that V_{0x} (the x-component of the velocity at t=0) = V_{fx} (the final x-component of the velocity at t=final). Given that we know that the acceleration in the x-direction is 0 (velocity is constant!), we can calculate R using the 2[nd] kinematic equation and substituting a=0: $\mathbf{X = V_0 t + \frac{1}{2} a t^2}$

Here, X represents the displacement of the projectile in the X direction, which we call R. V_0 is constant so $\mathbf{R = V_x t}$. But what is t? How long is the projectile flying in the air?

The Vertical Component V_y

While the x-component of the velocity is unhindered by air resistance, the velocity in the y-direction is subject to gravity! The projectile must eventually return to the surface! Using the first kinematic equation: $\mathbf{V_f = V_0 + at}$ where a=gravity (a negative acceleration), we can rearrange the signs to read $\mathbf{V_{fy} = V_{0y} - gt}$. Given our choice of axes and our approach to handle the Y component of motion independently from that of the X component, we can think of the Y component problem exactly as if we tossed a ball straight up into the air with velocity V_{0y}. When launched straight up, the projectile will rise until it's Y velocity reaches 0 at which point it will be at its maximum height. Using the 1st kinematic equation with $V_{fy} = 0$ and solving for t, we get that the time it will take for the projectile to reach its maximum height is $t_{maximum} = v_{0y}/g$. The projectile will then fall to the ground with a downward trip that mirrors its trip up. So, the total time the projectile is airborne is $2t_{maximum}$= $\mathbf{2v_{0y}/g}$

Putting it Together

From our analysis on the x-component, we calculated that the distance the projectile would fly in the X direction, called R, was found by **$R = V_x t$.** From our analysis on the y-component, we found that the flight time of the projectile was **$t = 2v_{0y}/g$.** Putting these results together yields that **R** = $2V_xV_{0y}/g$ = **$2(V_0^2/g)\sin\Theta\cos\Theta$.** You can simplify using the trigonometric identity of $2\sin\Theta\cos\Theta = \sin(2\Theta)$ to get **$R = (V_0^2/g)\sin(2\Theta)$.** Given that the SINE function yields values between -1 and +1, you can see that the value of R is maximized then $\sin(2\Theta) = 1$ or where Θ=45 degrees. Likewise, if the projectile is shot straight upwards into the sky (Θ = 90 degrees and 2Θ = 180 degrees), the value of R will be equal to 0 and the projectile will not travel any distance in the X direction (though it's initial launch speed in the Y direction is maximized, so overall flight time is maximized).

What is the maximum height achieved?

We calculated that the is **$t_{maximum} = v_{0y}/g$** and we can calculate the **$V_{0y} = V_0\sin(\Theta)$.** Additionally, using the 4th kinematic equatuin **$V_f^2 = V_0^2 + 2ad$** where d is this problem represents total vertical displacement H (total vertical distance traveled between t=0 and $t_{maximum}$). At the pinnacle of the projectiles flight, **$V_f^2 = 0$** and solving for d, d= $V_{0y}^2/(2g)$ = **$V_0\sin^2(\Theta)/(2g)$.**

<u>**Exercises**</u>

1. Which of the following statements are true of the vertical motion of projectile launched at an angle? (State the corrected statement if you find a statement False.)
   a. The vertical component of a projectile's velocity is a constant value of 9.8 m/s.
   b. The vertical component of a projectile's velocity is changing at a constant rate.
   c. The vertical velocity of a projectile is 0 m/s at the peak of its trajectory.
   d. The vertical velocity of a projectile is unaffected by the horizontal velocity; these two components of motion are independent of each other.
   e. The final vertical velocity of a projectile is always equal to the initial vertical velocity.
   f. The vertical acceleration of a projectile is 0 m/s^2 when it is at the peak of its trajectory.
   g. As a projectile rises towards the peak of its trajectory, the vertical acceleration will decrease; as it falls from the peak of its trajectory, its vertical acceleration will decrease.
   h. As a projectile rises towards the peak of its trajectory, the vertical acceleration is directed upward; as it falls from the peak of its trajectory, its vertical acceleration is directed downward.
   i. The peak height to which a projectile rises above the launch location is dependent upon the initial vertical velocity.
   j. As a projectile rises towards the peak of its trajectory, the vertical velocity will decrease; as it falls from the peak of its trajectory, its vertical velocity will increase. k. Consider a projectile launched from ground level at a fixed launch speed and a variable angle and landing at ground level. The vertical displacement of the projectile during the first half of its trajectory (i.e., the peak height) will always increase as the angle of launch is increased from 0 degrees to 90 degrees.

l. Consider a projectile launched from ground level at a fixed launch angle and a variable launch speed and landing at ground level. The vertical displacement of the projectile during the first half of its trajectory (i.e., the peak height) will always increase as the launch speed is increased.

2. A ball is thrown horizontally from the roof of a building that is 75meters tall with a speed of 5m/s. Evaluate the following (assuming $g=-9.8m/s^2$).
    a. How much time does it take for the ball to hit the ground?
    b. How far from the base of the building will the ball land?
    c. What is the velocity of the ball just as it reaches the ground? (hint: how do we combine the x & y component velocities?)
3. A projectile is launched from ground-level with an initial speed of 150m/s at an angle of 47 degrees above the horizon. (Please re-derive the formulas for the following as opposed to just plugging in values to the formulas) Determine the following quantities:
    a. Total airtime of the projectile
    b. Maximum height achieved by the projectile
    c. Horizontal distance covered by the projectile
4. When launched at a 40 degree angle, the highest barrier a projectile can clear is 20m. What was the launch speed of the projectile?
5. Two projectiles are fired at equal speeds but different angles. One is launched at 60 degrees and the other at 30 degrees. Which projectile covers more horizontal distance? Which projectile has a longer flight time?

Topic 4: Forces

A force is an interaction between two objects that tends to produce an acceleration in the objects. Acceleration occurs when there is a net force on an object. No acceleration occurs if the net force (the sum of all the forces) is equal to zero. In other words, acceleration occurs where there is a net force and no acceleration occurs when the forces are balanced. Remember that acceleration will cause a change in velocity to an object (either magnitude or direction or both). Isaac Newton studied forces and derived his Three Laws of Motion to describe their interactions.

Newton's First Law

Newton's first law states that an object at rest tends to remain at rest, and an object in motion tends to remain in motion with a constant velocity (constant speed and direction of motion), unless it is acted on by a nonzero net force.

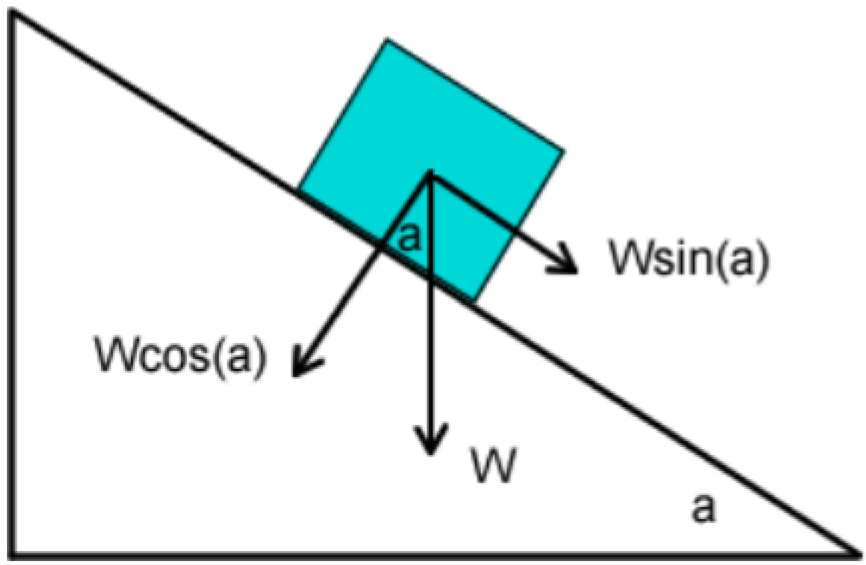
Newton's Second Law

If there is a net force acting on an object, the object will have an acceleration and the object's velocity will change. How much acceleration will be produced by a given force? Newton's second law states that for a particular force, the acceleration of an object is proportional to the net force and inversely proportional to the mass of the object. This can be expressed in the form of an equation: F=ma or Force = mass multiplied by acceleration.

Newton's Third Law

A force is an interaction between objects, and forces exist in equal-and-opposite pairs. Newton's third law summarizes this as follows: when one object exerts a force on a second object, the second object exerts an equal-and-opposite force on the first object. Note that "equal-and opposite" is the shortened form of "equal in magnitude but opposite in direction". Although the forces between two objects are equal-and-opposite, the effect of the forces may or may not be similar on the two; it depends on their masses. Remember that the acceleration depends on both force and mass, and let's look at the force exerted by the Earth on a falling object. If we drop a 100 g (0.1 kg) ball, it experiences a downward acceleration of 9.8 m/s^2, and a force of about 1 N, because it is attracted towards the Earth. The ball exerts an equal and opposite force on the Earth, so why doesn't the Earth accelerate upwards towards the ball? The answer is that it DOES, but because the mass of the Earth is so large (6.0 x 10^{24} kg) the acceleration of the Earth is much too small (about 1.67 x 10^{-25} m/s^2) for us to notice. In cases where objects of similar mass exert forces on each other, the fact that forces come in equal-and-opposite pairs is much easier to see.

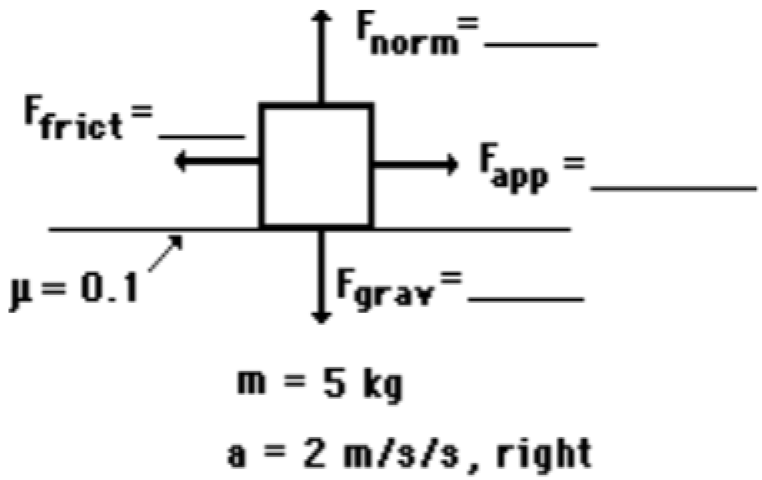
Inclined Planes and Resolution of Forces



In the above diagram, a green block with weight W sits **unmoving** on an incline plane with angle “a” relative the horizon. (Weight here is a force that is equal to an object’s mass multiplied by the gravity that it is subjected to. Weight is always directionally downwards towards the Earth’s surface). Why is the block not moving? Since the block is not moving and not accelerating, we can assume that the net of all forces acting on the block are balanced.

If we break up W into directional components relative to the surface of the inclined plane, we can see that the weight is pushing the block into the plane with a component force of Wcos(a). Given the system isn’t accelerating in that direction, we know there must be an equal but opposite force of magnitude Wcos(a) pushing back on the block from the plane. This force that is exerted by the plane perpendicular to the plane’s surface is known at the normal force N. Additionally, we can see that the weight is pulling the block down the plane with a component force of Wsin(a). Given the block is not accelerating down the plane, we know that there must be another equal but opposite force with magnitude Wsin(a) pulling against the blocks desired direction of motion. This force is the friction force that resists motion. The formula for friction is $F = N\mu$ where N is the normal force of the plane pushing back on the box perpendicularly to its surface and μ is a scalar called the coefficient of friction. Given we know the forces are balance and friction is $W\sin(a) = \mu N$ and $N= W\cos(a)$, we can see that solving for $\mu = \tan(a)$.

Force Diagram Example

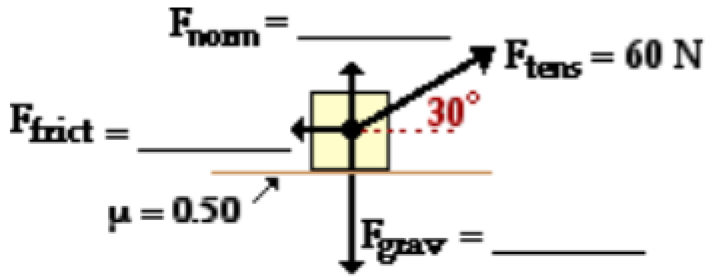


The diagram above is a force diagram that shows 4 forces on an object that rests on a flat plane. We can see that the mass of the object is 5kg, the coefficient of friction is 0.1 (scalar quantity), there is a force of friction that is counter to the direction of motion and a normal force that is equal to and opposite to the force of the object's weight on the surface.

We can see that the net acceleration is $2m/s^2$. Given the m=5kg and the equation F=ma, we know that there must be a net force of 10Newtons to the right to cause that acceleration. Additionally, there is no acceleration in the y-direction so we know that the Weight Force and Normal Force must be equivalent. The weight force = ma = $(5kg)(-9.8m/s^2)$ = -49Newtons directionally downwards. The Normal force then must be +49N upwards to negate the weight. In the x direction, we know that the friction force will be Nμ = (49N)(0.1) = 4.9N in the direction opposite to motion. Given that we know the system has a net force of 10N to the right, there must be an applied force of 14.9N to the right such that the X direction nets 10N and causes an acceleration in that direction.

Exercises

1. Which of the following are always true of an object that is at equilibrium? Include all that apply.
   a. All the forces acting upon the object are equal.
   b. The object is at rest.
   c. The object is moving and moving with a constant velocity.
   d. The object has an acceleration of zero. \
   e. There is no change in the object's velocity.
   f. The sum of all the forces is 0 N.
   g. All the forces acting upon an object are balanced



m = 8 kg

a = ________ m/s/s, right

ΣF = ________

2. In the above diagram, fill in the blanks. Assume that gravity is $10m/s^2$. The F_{tens} is an applied force on the system. Note that as the F_{tens} has an upwards y-component, the Normal force will not equal to the Weight force like in previous examples but the sum of all forces in the Y direction will balance.
3. A 945-kg car traveling rightward at 22.6 m/s slams on the brakes and skids to a stop (with locked wheels). If the coefficient of friction between tires and road is 0.972, determine the distance required to stop. Hint: the only force is the x-direction acting on the car is the force of friction. Use the kinematic equations to determine stopping distance.